Conclusions

The main objective of the WPSAC package is to provide accurate first sizing of wing structural elements at very early stages of the design process. The requirement for such a program like WPSAC becomes apparent considering the possible redesign problems as a result of the improperly conducted trade and initial sizing studies. WPSAC has been developed according to the modeling and stress analysis techniques employed at TAI.

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Analytical Solution for Wing Dihedral Effect

W. F. Phillips*

Utah State University, Logan, Utah 84322-4130

Nomenclature

a_n	=	coefficients in the classical infinite series solution to Prandtl's lifting line equation
b	=	wingspan
$C_{L,\alpha}$	=	lift slope for the complete finite wing
$\tilde{C}_{L,\alpha}$	=	local in situ section lift slope, including the effects of local wing downwash
$C_{\ell,\beta}$	=	roll stability derivative, that is, change in rolling moment coefficient with respect to sideslip angle
c	=	local wing section chord length
R_A	=	aspect ratio, b^2/S
S	=	wing planform area
V	=	airspeed
v	=	sideslip component of relative wind
y	=	spanwise coordinate
β	=	sideslip angle
Γ	=	wing dihedral angle
$(\Delta C_\ell)_\Gamma$	=	contribution to the rolling moment coefficient that results from sideslip combined with wing dihedral
$(\Delta C_{\ell,\beta})_\Gamma$	=	contribution of wing dihedral to the roll stability derivative
θ	=	change of variables defined by Eq. (5)

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*Professor, Mechanical and Aerospace Engineering Department. Member AIAA.

Introduction

FOR a conventional airplane, the change in rolling moment due to sideslip is heavily influenced by the dihedral angle of the wing. Wing dihedral affects the roll stability derivative of an airplane because it causes the lift on the right and left semispans to respond differently to sideslip. As shown in Fig. 1 for a wing with positive dihedral, sideslip produces an increase in angle of attack on the windward semispan and a decrease in angle of attack on the leaside semispan. When the sideslip component of relative wind is resolved into components parallel with and normal to each semispan, as shown in Fig. 1, the upward normal component is $v \sin\Gamma$ on the right semispan and $-v \sin\Gamma$ on the left semispan. For small aerodynamic angles, the sideslip velocity can be closely approximated as the product of the airspeed and sideslip angle. Likewise, the angle of attack is closely approximated as the upward normal component of relative wind divided by the airspeed. Thus, sideslip produces a change in angle of attack that can be closely approximated for the right and left semispans as $\beta \sin\Gamma$ and $-\beta \sin\Gamma$, respectively.

The differential in angle of attack between the right and left semispans creates a differential in lift, which in turn produces a rolling moment. With the standard sign convention, lift generated on the left semispan ($y < 0$) produces a positive rolling moment, whereas lift generated on the right semispan ($y > 0$) produces a negative rolling moment. Thus, the rolling moment arm for the lift on any wing section is $-y$. When the small angle approximations discussed earlier are used, the net contribution to the rolling moment coefficient that results from sideslip combined with wing dihedral can be written

$$(\Delta C_\ell)_\Gamma = -\frac{\beta \sin\Gamma}{Sb} \int_{y=-b/2}^{b/2} \tilde{C}_{L,\alpha} |y| c \, dy \tag{1}$$

where $\tilde{C}_{L,\alpha}$ includes the effects of local induced downwash. In general, the local induced downwash varies with spanwise position. Thus, $\tilde{C}_{L,\alpha}$ depends not only on the airfoil section geometry but also on the position that the section occupies along the span of the finite wing. Even if the section geometry and airfoil section lift slope are constant over the span of the wing, the in situ section lift slope can vary with spanwise position. Typically, the in situ section lift slope will vary from a maximum at the midspan to zero at the wingtips.

The local induced downwash and in situ section lift slope will be affected by the sideslip. Thus, the derivative of Eq. (1) with respect to β is

$$\frac{\partial}{\partial\beta}(\Delta C_\ell)_\Gamma = -\frac{\sin\Gamma}{Sb} \int_{y=-b/2}^{b/2} \tilde{C}_{L,\alpha} |y| c \, dy - \frac{\beta \sin\Gamma}{Sb} \int_{y=-b/2}^{b/2} \frac{\partial \tilde{C}_{L,\alpha}}{\partial\beta} |y| c \, dy$$

However, the roll stability derivative $C_{\ell,\beta}$ is evaluated at $\beta = 0$ and the contribution of wing dihedral to this aerodynamic derivative can be written

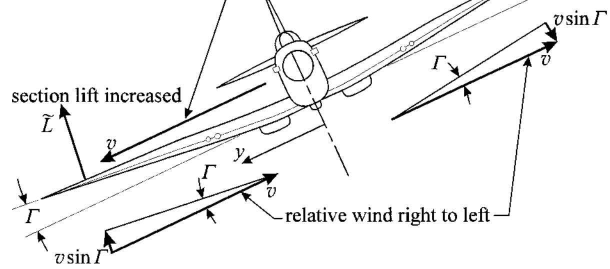

Fig. 1 Effect of sideslip and wing dihedral on local section lift.

$$(\Delta C_{\ell,\beta})_\Gamma = -\frac{\sin\Gamma}{Sb}\left[\int_{y=-b/2}^{b/2} \tilde{C}_{L,\alpha}|y|c\,\mathrm{d}y\right]_{\beta=0} \tag{2}$$

Because the integral in Eq. (2) is evaluated at $\beta = 0$, both the flow and the wing are symmetric about the midspan and Eq. (2) can also be written as

$$(\Delta C_{\ell,\beta})_\Gamma = -\frac{2\sin\Gamma}{Sb}\left[\int_{y=0}^{b/2} \tilde{C}_{L,\alpha}yc\,\mathrm{d}y\right]_{\beta=0} \tag{3}$$

The only difficulty in evaluating Eq. (3) is in knowing the in situ section lift slope as a function of y. One approach to evaluating the integral in Eq. (3) has been to assume a constant in situ section lift slope equal to the airfoil section lift slope.[1,2] This approximation overestimates the roll stability derivative because it overestimates the in situ section lift slope near the wingtips where the moment arm for roll is greatest. The other method that is commonly used to deal with the unknown in situ section lift slope in Eq. (3) is to use an empirical relation for the wing dihedral effect.[2–7] There is a third alternative, which appears to have been widely overlooked. That is the application of lifting line theory.

Lifting Line Solution

For subsonic airspeeds with low Mach number, Prandtl's lifting line theory[8,9] can be used to determine the in situ section lift slope needed to evaluate the integral in Eq. (3). For large dihedral angles, it is necessary to use a numerical lifting line solution[10] because the classical infinite series solution applies only to wings with no sweep or dihedral. However, for unswept wings in the limit as the dihedral angle approaches zero, the integral in Eq. (3) can be evaluated from the classical solution to Prandtl's lifting line equation.[11–13] For a wing of arbitrary planform shape having no geometric or aerodynamic twist, this solution predicts an in situ section lift slope of

$$\tilde{C}_{L,\alpha} = \frac{4b}{c}\sum_{n=1}^{\infty} a_n \sin(n\theta) \tag{4}$$

where θ is the change of variables given by

$$y = \tfrac{1}{2}b\cos\theta \tag{5}$$

From Eq. (5), we have

$$\mathrm{d}y = -\tfrac{1}{2}b\sin\theta\,\mathrm{d}\theta \tag{6}$$

Using Eqs. (4–6) in Eq. (3) with the definition of aspect ratio, after applying the trigonometric identity $\sin(2\theta) = 2\sin\theta\cos\theta$ and the small angle approximation $\sin\Gamma \cong \Gamma$, we obtain

$$(\Delta C_{\ell,\beta})_\Gamma = -R_A\Gamma\sum_{n=1}^{\infty} a_n \int_{\theta=0}^{\pi/2} \sin(n\theta)\sin(2\theta)\,\mathrm{d}\theta \tag{7}$$

The integral in Eq. (7) is readily evaluated to yield

$$\int_{\theta=0}^{\pi/2} \sin(n\theta)\sin(2\theta)\,\mathrm{d}\theta = \begin{cases} \dfrac{2\sin(n\pi/2)}{4-n^2}, & n \neq 2 \\ \pi/4, & n = 2 \end{cases}$$

After applying this result, Eq. (7) becomes

$$(\Delta C_{\ell,\beta})_\Gamma = -R_A\Gamma\left[\frac{2}{3}a_1 + \frac{\pi}{4}a_2 + \sum_{n=3}^{\infty}\frac{2\sin(n\pi/2)}{4-n^2}a_n\right] \tag{8}$$

Also from the classical lifting line solution, the lift slope for the complete finite wing is given by

$$C_{L,\alpha} = \pi R_A a_1$$

Thus, Eq. (8) can be conveniently rearranged as

$$(\Delta C_{\ell,\beta})_\Gamma = -(2\Gamma/3\pi)\kappa_\ell\, C_{L,\alpha} \tag{9}$$

where

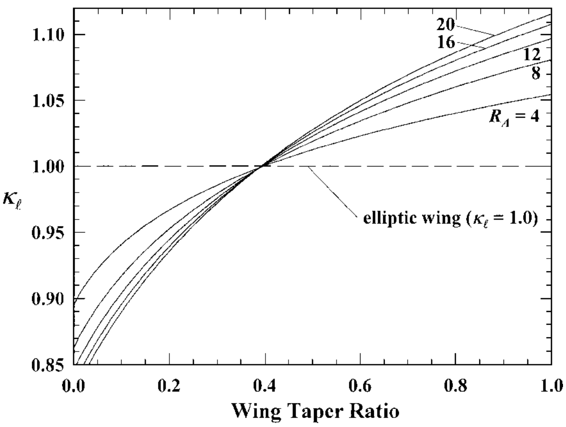

Fig. 2 Dihedral factor for a tapered wing with no geometric or aerodynamic twist.

$$\kappa_\ell = 1 + \frac{3\pi}{8}\frac{a_2}{a_1} + \sum_{n=3}^{\infty}\frac{3\sin(n\pi/2)}{4-n^2}\frac{a_n}{a_1} \tag{10}$$

and the coefficients in Eq. (10) are readily evaluated from the known planform geometry. Historically, these coefficients have usually been evaluated from collocation methods. Typically, the series is truncated to a finite series, and the coefficients in the finite series are evaluated by requiring the lifting line equation to be satisfied at a number of spanwise locations equal to the number of terms in the series. A very straightforward method was first presented by Glauert.[14] The most popular method, based on Gaussian quadrature, was originally presented by Multhopp.[15] Most recently, Rasmussen and Smith[16] have presented a more rigorous and more rapidly converging method, based on a Fourier series expansion similar to that first used by Lotz[17] and Karamcheti.[18]

Results

For the special case of an elliptic wing with no geometric or aerodynamic twist, all coefficients in the infinite series solution are zero, with the exception of a_1. Hence, the right-hand side of Eq. (10) is unity for an elliptic wing, and we see that κ_ℓ is the ratio of the roll stability derivative developed by a wing of arbitrary planform to that developed by an elliptic wing having the same dihedral and lift slope. Note that, because the lift slope for an elliptic wing is greater than that for a wing of any other planform with the same aspect ratio, the two wings that define the ratio for κ_ℓ do not have the same aspect ratio. Figure 2 shows how κ_ℓ varies with aspect ratio and taper ratio for a tapered wing with no geometric or aerodynamic twist.

Conclusions

Strictly speaking, Eqs. (9) and (10) apply only at low Mach numbers to wings with no sweep. However, the contribution of wing dihedral to the roll stability derivative results directly from the lift developed on the wing. Thus, higher subsonic Mach numbers and small sweep angles should affect both lift and wing dihedral contributions to the roll stability derivative in nearly the same proportion. With this line of reasoning, Eqs. (9) and (10) could be applied with caution to higher Mach numbers and wings with some sweep, provided that the actual lift slope for the swept wing at the actual flight Mach number is used in Eq. (9).

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Viscous Compressible Flow Through a Hole in a Plate, Including Entry Effects

P. M. Galluzzo* and H. Babinsky†
University of Cambridge,
Cambridge, England CB2 1PZ, United Kingdom
and
G. R. Inger‡
Iowa State University, Ames, Iowa 50011-3231

Nomenclature

A	=	hole cross-sectional area
A_{plate}	=	total surface area of porous plate
D	=	nominal circular diameter of hole
k_D	=	scaling factor for diameter of each hole, due to roughness (≤ 1)
$k_{porosity}$	=	scaling factor for the porosity of each plate, due to blocked holes (≤ 1)
ℓ_i	=	entry length, defined as distance along the axis of a hole where the center-line velocity reaches 99% of its fully developed value
M	=	local Mach number in hole
$\dot{m}$	=	total transpiration mass flow through porous plate
p	=	static pressure
p_0	=	stagnation pressure
R	=	ideal gas constant
Re	=	hole Reynolds number based on diameter
Re_{eff}	=	effective Reynolds number of flow inside the hole
T	=	absolute static temperature, assumed constant along hole
t	=	plate thickness
V	=	cross sectionally averaged flow velocity in hole
x	=	axial coordinate inside hole
γ	=	specific heat ratio
Δp_{bubble}	=	pressure drop across the separation bubble at hole entrance
κ	=	ratio of apparent skin-friction coefficient in entry length to the skin-friction coefficient of fully developed pipe flow
μ	=	absolute viscosity
ρ	=	cross sectionally averaged density
$\bar{\tau}_w$	=	apparent wall skin friction, averaged over axial length

Subscripts

1	=	hole entrance station
2	=	hole exit station

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*Graduate Research Student, Department of Engineering.

†Senior Lecturer, Department of Engineering. Member AIAA.

‡Professor, Department of Aerospace Engineering and Engineering Mechanics. Associate Fellow AIAA.

Introduction

WALL transpiration through porous surfaces is used in many aerodynamic flow control devices as a means of influencing boundary-layer behavior. The use of plates with microscopic laser-drilled holes has come to be preferred for this purpose. To predict the performance of such plates one must have an accurate knowledge of how the transpiration mass flux depends on relevant parameters such as pressure difference, hole diameter, and fluid properties. The transpiration mass flux is related to the fluid velocity inside each hole as

$$\dot{m} = (\rho_2 V_2) \times A_{plate} \times \text{porosity} \tag{1}$$

Inger and Babinsky[1] have recently proposed a new theory to predict the mass flux vs pressure difference relationship, based on a solution of the continuity equation, the momentum equation, and the ideal gas equation, for each hole independently, that is, a large distance is assumed to exist between holes.

Near the entrance to each hole of a porous plate, the flow is not yet fully developed, and the velocity gradient near the walls is quite steep. This will increase the skin friction near the wall and, by altering the velocity profile of the flow, cause a change in the momentum of the flow. Inger and Babinsky[1] assumed that the average value of skin friction over the entry length ℓ_i is some constant κ greater than its fully developed value of $(16/Re) \times \frac{1}{2}\rho V^2$ and that the average value of skin friction over the entire length of the hole is given by

$$\bar{\tau}_w \Big/ \tfrac{1}{2}\rho V^2 = (16/Re)[(t - \ell_i)/t] + \kappa(16/Re)(\ell_i/t)$$
$$= (16/Re)[1 + (\kappa - 1)(\ell_i/t)] \tag{2}$$

Inger and Babinsky[1] proposed that the value of κ be chosen empirically to match experimental data. They assumed also that by manipulating κ in this way the effects of irregularities in the holes of a porous plate, such as roughness and sharp-edged intakes, could be accommodated for. However, it will be shown here that an analytical value for κ may be calculated for nearly all cases and that nonideal aspects of the hole geometry can be dealt with separately.

Proposed Improvements to Skin-Friction Model

When the flow enters each hole it is not yet fully developed; initially, a boundary layer develops along the wall of the hole, causing the skin friction to be larger than the value for fully developed pipe flow by some factor κ [Eq. (2)]. Inger and Babinsky[1] recommended a value for κ of between 2 and 3, giving κ a dual role: On one hand, it was designed to capture entry effects; on the other hand, it was